Large Structural VARs with Multiple Sign and Ranking Restrictions

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Abstract

Large VARs are increasingly used in structural analysis as a unified framework to study the impacts of multiple structural shocks simultaneously. However, the concurrent identification of multiple shocks using sign and ranking restrictions poses significant practical challenges to the point where existing algorithms cannot be used with such large VARs. To address this, we introduce a new numerically efficient algorithm that facilitates the estimation of impulse responses and related measures in large structural VARs identified with a large number of structural restrictions on impulse responses. The methodology is illustrated using a 35-variable VAR with over 100 sign and ranking restrictions to identify 8 structural shocks.

Keywords: large vector autoregression, sign restriction, ranking restriction, shrinkage prior

JEL classifications: C11, C55, E50

1 Introduction

Vector autoregressions (VARs) are a workhorse model in macroeconomic forecasting and structural analysis. Among the many methodological advances in the structural VAR (SVAR) literature since the pioneering work by Sims (1980), two recent developments are the most prominent. First, there is a growing recognition of the need to exploit more information in structural analyses, motivated by the concern that informational deficiency (using an information set that is too small relative to that of economic agents) substantially distorts estimates of impulse responses and related objects (Hansen and Sargent, 1991; Lippi and Reichlin, 1993, 1994). Starting from the seminal paper by Leeper, Sims, and Zha (1996) that develops various medium-sized structural VARs to study the effects of monetary policy, large VARs with dozens of endogenous variables are increasingly being used in applications. This trend gained momentum after the influential work by Bańbura, Giannone, and Reichlin (2010), who demonstrate the benefits of including a large number of variables for both forecasting and structural analysis. Notable applications using large VARs include Carriero, Kapetanios, and Marcellino (2009), Koop (2013), Ellahie and Ricco (2017) and Crump, Eusepi, Giannone, Qian, and Sbordone (2021).

The second development relates to the methods for identifying structural shocks. More specifically, there has been a gradual departure from conventional recursive or zero restrictions to alternative structural restrictions that are deemed to be more credible. An important class of identifying restrictions imposes sign restrictions motivated by economic theory, developed in a series of papers by Faust (1998), Canova and Nicolo (2002) and Uhlig (2005). Extensions of this identification approach, such as ranking restrictions proposed in Amir-Ahmadi and Drautzburg (2021), are also widely used in empirical work.

The convergence of these two developments naturally requires the estimation of large structural VARs identified by imposing sign and ranking restrictions on the impulse responses. However, this remains practically infeasible in high-dimensional settings. For instance, using the popular accept-reject algorithm of Rubio-Ramirez, Waggoner, and Zha (2010) to impose sign restrictions might take days in larger-scale applications. Thus, this computational burden severely limits the use of these more credible restrictions in large systems.

We develop a new approach to estimate large SVARs identified using a large number of structural restrictions on impulse responses, which was until now computationally infeasible. The new algorithm builds upon the accept-reject algorithm of Rubio-Ramirez, Waggoner, and Zha (2010), which we now briefly describe to provide some perspective. First, given a uniformly drawn orthogonal matrix (i.e., a matrix drawn according to the Haar measure), Rubio-Ramirez, Waggoner, and Zha (2010) check if the implied impulse responses satisfy all restrictions. If all the restrictions are satisfied (the draw is admissible), accept the draw and the implied impulse responses; otherwise, obtain another uniform draw and repeat the procedure. The main computational bottleneck of this algorithm comes from the fact that in high-dimensional settings with a large number of structural restrictions, it is highly unlikely that any orthogonal matrix drawn uniformly is admissible. Consequently, one typically needs to sample a huge number of orthogonal matrices to obtain one that is admissible.

The key idea of our proposed algorithm comes from the recognition that, given a uniformly distributed orthogonal matrix, a vast collection of uniform draws can be constructed by permuting its columns and switching the signs of the columns.[1] More importantly, all these obtained orthogonal matrices are equivalent, in the sense that they represent exactly the same structural shocks of the original orthogonal matrix, after relabeling the shocks and proper sign normalizations. Additionally, one can effectively search through this collection to locate any members that satisfy all structural restrictions with trivial computations. In this way, the new algorithm significantly increases the probability of obtaining an admissible draw with virtually no additional costs. In our benchmark setting, we impose that any identification restriction is only imposed on impact to allow for fast checking of identification restrictions. Economic theory generally only produces robust restrictions *across theoretical models* only on impact, giving a justification for this approach. However, we also discuss how to extend our approach to sign restrictions at longer horizons as well as ranking restrictions along the lines of Graeve and Karas (2014) and Amir-Ahmadi and Drautzburg (2021).

To illustrate our proposed algorithm, we consider three applications, two empirical applications based on US data and one set of Monte Carlo simulations. First, we estimate a 15-variable VAR with more than 40 sign and ranking restrictions to identify 5 structural shocks based on empirical applications in Furlanetto, Ravazzolo, and Sarferaz (2019) and

[1] Since the Haar measure is invariant under permutations and sign switches, any member of this collection is also uniformly distributed in the orthogonal group.

Chan (2022). As a benchmark, we use the algorithm of Rubio-Ramirez, Waggoner, and Zha (2010) to uniformly draw orthogonal matrices from the admissible set and compute implied impulse responses. It takes about 3.6 billion orthogonal matrices to obtain 1,000 admissible draws, and the estimation takes about 6 days on a standard desktop. In contrast, the new algorithm requires only about 31,000 orthogonal matrices to obtain 1,000 admissible draws, and the entire exercise takes about 16 seconds. We also confirm empirically that both algorithms give identical impulse responses. Second, we conduct a series of Monte Carlo simulations to illustrate the empirical performance of the proposed method, and show that it works well even in settings with large numbers of variables and structural shocks.

Our second empirical application considers a larger 35-variable VAR with over 100 sign and ranking restrictions to identify 8 structural shocks: demand, investment, financial, monetary policy, government spending, technology, labor supply and wage bargaining. These macroeconomic and financial variables are broadly similar to those of Crump, Eusepi, Giannone, Qian, and Sbordone (2021) and are closely monitored by policy institutions and market participants. Our high-dimensional model provides a unified framework to study the impacts of multiple structural shocks simultaneously. In particular, this framework allows us to disentangle the impacts of different types of demand and supply shocks on key macroeconomic variables. Even for such a large system, the estimation takes only 14 minutes. Therefore, this application demonstrates that it is practical to study the impacts of multiple structural shocks jointly in a large system using the proposed approach.

Our paper contributes to the emerging literature on efficient methods for conducting structural analysis using large VARs. As noted in Crump, Eusepi, Giannone, Qian, and Sbordone (2021), central banks and policy institutions routinely monitor and forecast dozens of key macroeconomic variables, and VARs provide a convenient framework for studying the joint impacts of multiple structural shocks. To reduce the computational burden of performing structural analysis in large systems, some recent papers, such as Korobilis (2022) and Chan, Eisenstat, and Yu (2022), propose using a factor model for the reduced-form VAR errors and structural identification restrictions are placed on factor loadings. In contrast, our paper uses a standard VAR framework where structural shocks are related to the reduced-form errors through an impact matrix. Therefore, the proposed algorithm is directly applicable to a wide variety of VARs currently used for structural analysis.

This paper also relates to the literature on efficient posterior sampling in structural VARs with informative priors on impulse responses (see, e.g., Kociecki, 2010; Baumeister and Hamilton, 2015, 2018). In particular, for VARs identified using sign restrictions, the proposed algorithm can be used in the first stage to generate proposal draws for an importance sampler to explore the posterior distribution that incorporates prior information on impulse responses; a recent example of such an importance sampler is given in Bruns and Piffer (2023). The proposed algorithm can thus boost the efficiency of the second-stage importance sampler and make it applicable beyond medium-sized models.

The remainder of this paper is organized as follows. Section 2 first outlines the identification of shocks in a structural VAR using sign restrictions. We then introduce the proposed algorithm for generating uniform draws of the impact matrix that satisfy all the sign restrictions at impact. Finally, we discuss how the proposed algorithm can be extended to handle other commonly-used identification schemes. Section 3 considers an illustration using a 15-Variable VAR with sign restrictions to identify 5 structural shocks. We compare the speed of the proposed algorithm as well as the impulse response estimates with those obtained using the algorithm of Rubio-Ramirez, Waggoner, and Zha (2010). Section 4 considers an application that involves 35 US macroeconomic and financial variables. We use over 100 sign and ranking restrictions to identify 8 structural shocks. Lastly, Section 5 concludes and outlines some future research directions.

2 Identification of Structural Shocks

In this section, we first outline the identification of structural shocks in a structural VAR using sign restrictions. In Section 2.1 we then introduce the proposed algorithm to efficiently generate draws of the impact matrix that satisfy all the sign restrictions at impact. Section 2.2 further discusses how the proposed algorithm can be extended to handle other commonly used identification schemes, such as ranking restrictions.

To set the stage, let $\mathbf{y}_t = (y_{1,t}, \dots, y_{n,t})'$ be an $n \times 1$ vector of endogenous variables that is observed over the periods $t = 1, \ldots, T$. Consider the following VAR with p lags:

$$\mathbf{y}_t = \mathbf{a}_0 + \mathbf{A}_1 \mathbf{y}_{t-1} + \cdots + \mathbf{A}_p \mathbf{y}_{t-p} + \mathbf{u}_t, \tag{1}$$

$$\mathbf{u}_t = \mathbf{B}_0 \mathbf{v}_t, \quad \mathbf{v}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_n), \tag{2}$$

where the vector of structural shocks $\mathbf{v}_t$ is related to the reduced-form errors $\mathbf{u}_t$ via the impact matrix $\mathbf{B}_0$ that is assumed to be non-singular. It follows that the covariance matrix of $\mathbf{u}_t$ is $\mathbf{\Sigma} \equiv \mathbf{B}_0 \mathbf{B}_0'$.

One main goal of estimating the VAR in (1)-(2) is to study the impact of structural shock $v_{j,t}$ on the endogenous variable $y_{i,t}$, $i = 1, \ldots, n$ and $j = 1, \ldots, m$. Specifically, the impulse response at horizon h is defined to be the expected change in the conditional mean of $y_{i,t+h}$ from the j-th structural shock $v_{j,t}$:

$$f_{i,j,h} = \mathbb{E}\left[y_{i,t+h} \,|\, \mathbf{v}_t = \mathbf{e}_j; \mathbf{B}_0, \mathbf{A}\right] - \mathbb{E}\left[y_{i,t+h} \,|\, \mathbf{v}_t = \mathbf{0}; \mathbf{B}_0, \mathbf{A}\right], \tag{3}$$

where $\mathbf{e}_j$ is the j-th column of the n-dimensional identity matrix $\mathbf{I}_n$ and $\mathbf{A} = (\mathbf{a}_0, \mathbf{A}_1, \ldots, \mathbf{A}_p)'$ is the $k \times n$ matrix of VAR coefficients with $k = np + 1$. Note that each impulse response $f_{i,j,h}$ depends implicitly on the impact matrix $\mathbf{B}_0$ and the VAR coefficients $\mathbf{A}$.

It is well known that under the setup in (1)-(2), $\mathbf{B}_0$ is not point-identified: since given any orthogonal matrix $\mathbf{Q} \in \mathbf{O}(n)$ and $\widetilde{\mathbf{B}}_0 = \mathbf{B}_0 \mathbf{Q}$, we have $\widetilde{\mathbf{B}}_0 \widetilde{\mathbf{B}}_0' = \mathbf{\Sigma}$. In other words, there is a range of impulse responses of variables to structural shocks, even if we fix the identifiable model parameters $(\mathbf{A}, \mathbf{\Sigma})$. One often proceeds by restricting the set of impulse responses—e.g., by imposing economically meaningful restrictions on the impulse responses. Starting from the influential papers by Faust (1998), Canova and Nicolo (2002) and Uhlig (2005), one prominent approach is to impose sign restrictions motivated by economic theory on the impulse responses.

More specifically, let $s_{i,j,h} \in \{-1, 0, 1\}$. Then, a sign restriction on the impulse response $f_{i,j,h}$ can be written as

$$s_{i,j,h} \times f_{i,j,h} \geqslant 0. \tag{4}$$

For example, if $s_{i,j,h} = 1$, then this sign restriction implies that the h-step-ahead response of the i-th variable to the j-th structural shock is restricted to be non-negative. If $s_{i,j,h} = 0$, then the sign restriction is not imposed on this response. We define the sign restrictions set $\mathcal{S}$ to be the collection of $s_{i,j,h}$ for all i, j, h.

Now, we can formally define the admissible set with respect to the set of sign restrictions $\mathcal{S}$ and model parameters $(\mathbf{A}, \mathbf{\Sigma})$:

$$\mathcal{Q}(\mathbf{A}, \mathbf{\Sigma}, \mathcal{S}) = \{\mathbf{Q} : \mathbf{Q} \in \mathbf{O}(n) \text{ and the impulse responses implied by } \mathbf{Q} \text{ and } (\mathbf{A}, \mathbf{\Sigma}) \text{ satisfy the restrictions in } \mathcal{S}\}.$$

A popular algorithm to obtain draws uniformly from the admissible set $\mathcal{Q}(\mathbf{A}, \mathbf{\Sigma}, \mathcal{S})$ is given in Rubio-Ramirez, Waggoner, and Zha (2010). It is an accept-reject algorithm and is implemented as follows. First, obtain a draw $\mathbf{Q}$ uniformly from the orthogonal group $\mathbf{O}(n)$ (i.e., according to the Haar measure). Then, set $\mathbf{R} = \mathbf{LQ}$, where $\mathbf{L}$ is the lower triangular Cholesky factor of $\mathbf{\Sigma}$. If the impulse responses implied by $(\mathbf{A}, \mathbf{R})$ satisfy all the restrictions in $\mathcal{S}$, then we accept $\mathbf{Q}$ (it is easy to see that $\mathbf{Q} \in \mathcal{Q}(\mathbf{A}, \mathbf{\Sigma}, \mathcal{S})$); otherwise, we obtain another draw uniformly from $\mathbf{O}(n)$ and repeat the procedure.

This algorithm is flexible and easy to implement and works well for a wide range of applications using small VARs. When the application requires a VAR that involves more than a dozen variables and restrictions, this algorithm tends to be computationally intensive, as it requires a large number of uniform draws from $\mathbf{O}(n)$ to get each draw from the admissible set $\mathcal{Q}(\mathbf{A}, \mathbf{\Sigma}, \mathcal{S})$. When n is large, this approach is simply computationally infeasible.

2.1 A New Algorithm

For high-dimensional systems with a large number of sign restrictions, it is highly unlikely that any given uniform draw from $\mathbf{O}(n)$, denoted as $\mathbf{Q} \sim \mathcal{U}(\mathbf{O}(n))$, would imply impulse responses that satisfy all the restrictions in $\mathcal{S}$. To make progress, we assume that $\mathcal{S} = \mathcal{S}_0$ where $\mathcal{S}_0$ collects sign restrictions that restrict only the signs of impulse responses at impact, i.e., $\mathcal{S}_0 = \{s_{i,j,0} : s_{i,j,0} \in \mathcal{S}\}$. There are two key reasons to focus on the subset $\mathcal{S}_0$. First, there is often a strong consensus in economic theory about the signs of impulse responses at impact but not at longer horizons (see, e.g., Canova and Paustian, 2011). Second, verifying the sign restrictions on impulse responses at impact is equivalent to verifying the signs of the elements in $\mathbf{R}$, where $\mathbf{R} = \mathbf{LQ}$ and $\mathbf{L}$ is the (lower triangular) Cholesky factor of $\mathbf{\Sigma}$. As such, this verification can be done very quickly without computing impulse responses at horizons larger than $h = 0$, which are much more costly to compute in large systems. Given a uniform draw $\mathbf{Q}$, one can build a huge collection of equivalent draws (defined below) and search through this collection to obtain any members that satisfy all sign restrictions with trivial computations.

More specifically, given $\mathbf{Q} \sim \mathcal{U}(\mathbf{O}(n))$, let $\mathcal{E}(\mathbf{\Sigma}, \mathbf{Q})$ denote the set

$$\mathcal{E}(\mathbf{\Sigma}, \mathbf{Q}) = \{\mathbf{E} : \mathbf{E} = \mathbf{LQPD}, \text{where } \mathbf{L} \text{ is the Cholesky factor of } \mathbf{\Sigma}, \mathbf{P} \text{ is an } n\text{-dimensional permutation matrix and } \mathbf{D} \text{ is a diagonal matrix with elements } \pm 1\}.$$

In other words, $\mathcal{E}(\mathbf{\Sigma}, \mathbf{Q})$ consists of all the permutations and sign switches of the columns of $\mathbf{LQ}$. Since there are $n!$ permutation matrices of dimension n and 2^n ways to construct an n vector from the two values ± 1, the cardinality of $\mathcal{E}(\mathbf{\Sigma}, \mathbf{Q})$ is $2^n n!$.

There are three key reasons to consider the set $\mathcal{E}(\mathbf{\Sigma}, \mathbf{Q})$. First, since each column of $\mathbf{LQ}$ can be viewed as the responses of the endogenous variables to a particular structural shock at impact, $\mathcal{E}(\mathbf{\Sigma}, \mathbf{Q})$ includes all possible permutations and sign normalizations of the structural shocks represented by $\mathbf{Q}$. That is, any member in $\mathcal{E}(\mathbf{\Sigma}, \mathbf{Q})$ represents exactly the same structural shocks as $\mathbf{Q}$—after relabeling the shocks and proper sign normalizations. Second, for any fixed $\mathbf{P}$ or $\mathbf{D}$ (respectively, a permutation matrix and a diagonal matrix with elements ± 1), it is orthogonal. Therefore, the Haar measure is invariant under right multiplication of $\mathbf{P}$ and $\mathbf{D}$. Hence, $\mathbf{QPD}$ is a uniform draw from $\mathbf{O}(n)$. Third, one can efficiently search through all the elements—all $2^n n!$ of them—in $\mathcal{E}(\mathbf{\Sigma}, \mathbf{Q})$ to find those that satisfy all the restrictions in $\mathcal{S}_0$ (discussed below). Put differently, given each $\mathbf{Q} \sim \mathcal{U}(\mathbf{O}(n))$, we automatically obtain $2^n n!$ economically equivalent candidates with trivial additional computations. For $n = 10$, the number of orthogonal matrices that we sort through is about 3.7 billion. When $n = 30$, the number is about 2.85×10^{41}.

To distinguish two structural shocks, we require that they have signed impacts on at least two common endogenous variables. In addition, their impacts on one variable have the same sign, while their impacts on the other variable have opposite signs. More formally, we assume that $\mathcal{S}_0$ satisfies the following assumption:

Assumption 1. For any $j \neq k$, $j, k = 1, \ldots, m$, there exist i_1 and i_2 such that $s_{i_1,j,0} =$

$s_{i_1,k,0} \neq 0$ and $s_{i_2,j,0} = -s_{i_2,k,0} \neq 0$.

Next, we describe an efficient way to go through all the elements in $\mathcal{E}(\mathbf{\Sigma}, \mathbf{Q})$ to locate those that satisfy all the restrictions in $\mathcal{S}_0$. Suppose that we have n endogenous variables and we are interested in m structural shocks. Let $\mathbf{T}$ denote an $m \times n$ matrix such that T_{ji}, the (j, i) element, is $+1$ if the i-th column of $\mathbf{R} = \mathbf{LQ}$ satisfies all the restrictions in $\mathcal{S}_0$ corresponding to j-th structural shock. If the negative of the i-th column of $\mathbf{R}$ satisfies all the inequalities in $\mathcal{S}_0$ corresponding to j-th structural shock, set $T_{ji} = -1$; otherwise $T_{ji} = 0$. In other words, the j-th row of $\mathbf{T}$ encodes all potential candidates among the columns of $\mathbf{R}$ that can represent the j-th structural shocks (those have entries ± 1). Therefore, if any row of $\mathbf{T}$ contains all 0, then none of the elements in $\mathcal{E}(\mathbf{\Sigma}, \mathbf{Q})$ satisfies all the restrictions in $\mathcal{S}_0$. In addition, by Assumption 1, each column of $\mathbf{T}$ has at most one $+1$ or -1—i.e., each column of $\mathbf{R}$ can satisfy (or violate) all the restrictions of at most one structural shock.

It is important to note that to compute the matrix $\mathbf{T}$, we only need to check each column of $\mathbf{R}$ to see if all the relevant inequalities are all satisfied, all violated or neither, for each structural shock $j = 1, \ldots, m$. Hence, it involves at most checking mn^2 inequalities to construct $\mathbf{T}$, which can be done quickly.

Let $\mathcal{E}(\mathbf{\Sigma}, \mathbf{Q}, \mathcal{S}_0)$ denote the subset of elements in $\mathcal{E}(\mathbf{\Sigma}, \mathbf{Q})$ that satisfy all restrictions in $\mathcal{S}_0$. Given the matrix $\mathbf{T}$, we can first determine whether or not $\mathcal{E}(\mathbf{\Sigma}, \mathbf{Q}, \mathcal{S}_0)$ is empty. If it is not, we then uniformly obtain an element from it as follows. Since each row of $\mathbf{T}$ contains at least one $+1$ or -1, for each $j = 1, \ldots, m$, we uniformly pick a column that has entries $+1$ or -1, say, i_j. And since each column contains at most one $+1$ or -1, we would not pick the same column twice. Given the sampled $i_1, \ldots, i_m$, we can reconstruct the element in $\mathcal{E}(\mathbf{\Sigma}, \mathbf{Q}, \mathcal{S}_0)$ that satisfies all the restrictions in $\mathcal{S}_0$. We summarize this algorithm in Algorithm 1.

Proposition 1. Under Assumption 1, the output $\widetilde{\mathbf{L}}$ from Algorithm 1 represents structural shocks that satisfy all the restrictions in $\mathcal{S}_0$. In addition, $\widetilde{\mathbf{L}} = \mathbf{LQ}^*$ for some $\mathbf{Q}^* \sim \mathcal{U}(\mathbf{O}(n))$ and $\widetilde{\mathbf{L}}\widetilde{\mathbf{L}}' = \mathbf{LL}' = \mathbf{\Sigma}$.

In other words, the proposed algorithm returns structural shocks that satisfy all the restrictions in $\mathcal{S}_0$ using a uniform draw $\mathbf{Q}^*$ from the orthogonal group $\mathbf{O}(n)$ such that $\widetilde{\mathbf{L}} = \mathbf{LQ}^*$. The proof of the proposition is given in Appendix A.

Algorithm 1 A new accept-reject algorithm to uniformly draw from the admissible set.

Given the posterior draws $\mathbf{A}$ and $\mathbf{\Sigma}$, obtain the lower triangular Cholesky factor $\mathbf{L}$ of $\mathbf{\Sigma}$ such that $\mathbf{\Sigma} = \mathbf{L}\mathbf{L}'$. Then, complete the following steps.

1. Sample $\mathbf{Q} \sim \mathcal{U}(\mathbf{O}(n))$. This can be done by sampling $\mathbf{Z} = (Z_{ij})$, where Z_{ij} are iid $\mathcal{N}(0,1)$ random variables, and returning the orthogonal matrix $\mathbf{Q}$ from the QR decomposition of $\mathbf{Z}$.

2. Given $\mathbf{L}$ and $\mathbf{Q}$, construct $\mathbf{R} = \mathbf{L}\mathbf{Q}$ and the associated $m \times n$ matrix $\mathbf{T}$.

3. If any row of $\mathbf{T}$ contains all 0, then go back to Step 1 and obtain another draw $\mathbf{Q}$; otherwise, let $\widetilde{\mathbf{L}}$ be an $n \times n$ zero matrix and complete the following steps:

   (a) For $j = 1, \ldots, m$, construct the index set $S_j = \{i : T_{ji} = +1 \text{ or } T_{ji} = -1, i = 1, \ldots, n\}$ and sample an element uniformly from S_j, denoted as, i_j. If $T_{ji_j} = +1$, set the j-th column of $\widetilde{\mathbf{L}}$ as the i_j-th column of $\mathbf{R}$; if $T_{ji_j} = -1$, set the j-th column of $\widetilde{\mathbf{L}}$ as the negative of the i_j-th column of $\mathbf{R}$.

   (b) For $j = m+1, \ldots, n$, let $S_j = \{1, \ldots, n\} \backslash \{i_1, \ldots, i_{j-1}\}$ and sample an element uniformly from S_j, denoted as, i_j. With probability 1/2, set the j-th column of $\widetilde{\mathbf{L}}$ as the i_j-th column of $\mathbf{R}$; otherwise set it as the negative of the i_j-th column of $\mathbf{R}$.

4. Return $\widetilde{\mathbf{L}}$, which represents structural shocks that satisfy all the restrictions in $\mathcal{S}_0$.

2.2 Extensions

In this section we discuss how the proposed algorithm can be extended to handle some other commonly-used identification schemes. We start with the ranking restrictions of Amir-Ahmadi and Drautzburg (2021). In particular, consider the ranking restriction of the form $s_{i,j,k,l} f_{i,j,0} \geqslant s_{i,j,k,l} \lambda_{i,j,k,l} f_{k,l,0}$ for $s_{i,j,k,l} \in \{-1, 0, 1\}$ and $\lambda_{i,j,k,l} \geqslant 0$, where $f_{i,j,0}$ is the impulse response of the i-th variable from the j-th structural shock on impact, as defined in (3).

For example, if $i = k$, $s_{i,j,k,l} = 1$ and $\lambda_{i,j,k,l} = 1$, then this ranking restriction implies that the impact of the j-th structural shock on the i-th variable is at least as large as the impact of the l-th shock on the same variable. On the other hand, if $j = l$, $s_{i,j,k,l} = 1$ and $\lambda_{i,j,k,l} = 1$, then this ranking restriction implies that the response of the i-th variable to the j-th structural shock at least as large as the response of the k-th variable to the same shock. Furthermore, it is easy to see that the ranking restriction includes the sign restriction as a special case by setting $\lambda_{i,j,k,l} = 0$.

Let $\mathcal{R}_0 = \{(s_{i,j,k,l}, \lambda_{i,j,k,l}) : i, k = 1, \dots, n, j, l = 1, \dots, m\}$ denote the ranking restrictions set on impact. We first consider the case where each ranking restriction involves only an individual structural shock (i.e., for $j \neq l$, $\lambda_{i,j,k,l} = 0$); the general case will be discussed afterward. In addition, to ensure that the structural shocks are distinct, we impose some regularity conditions on $\mathcal{R}_0$. Intuitively, to distinguish two structural shocks, we require that either 1) they have signed impacts on at least two common endogenous variables, where on one variable they have the same sign and on the other they have opposite signs; or 2) the impact on a linear combination of two variables from one shock is positive whereas that from the other shock is negative. Formally, we assume $\mathcal{R}_0$ satisfies the following assumption:

Assumption 2. For any $j \neq l$, $j, l = 1, \dots, m$, at least one of the following conditions hold:

1. there exist i_1 and i_2 such that $s_{i_1,j,k_1,m_1} = s_{i_1,l,k_2,m_2} \neq 0$ and $s_{i_2,j,k_3,m_3} = -s_{i_2,l,k_4,m_4} \neq 0$ for some $k_1, k_2, k_3, k_4, m_1, m_2, m_3, m_4$, with $\lambda_{i_1,j,k_1,m_1} = \lambda_{i_1,l,k_2,m_2} = \lambda_{i_2,j,k_3,m_3} = \lambda_{i_2,l,k_4,m_4} = 0$;
2. there exist i_1 and i_2 such that $s_{i_1,j,i_2,j} = -s_{i_1,l,i_2,l} \neq 0$ and $\lambda_{i_1,j,i_2,j}, \lambda_{i_1,l,i_2,l} > 0$.

Condition 1 in Assumption 2 is essentially an extension of Assumption 1 to the case of ranking restrictions. For example, if $s_{i_1,j,k_1,j} = s_{i_1,l,k_2,l} = 1$ and $s_{i_2,j,k_3,j} = -s_{i_2,l,k_4,l} = 1$, then Condition 1 implies $f_{i_1,j,0} \geqslant 0$, $f_{i_1,l,0} \geqslant 0$, $f_{i_2,j,0} \geqslant 0$, $f_{i_2,l,0} \leqslant 0$. Condition 2 discriminates the two structural shocks by their different signed impacts on a linear combination of two variables. For instance, if $s_{i_1,j,i_2,j} = -s_{i_1,l,i_2,l} = \lambda_{i_1,j,i_2,j} = \lambda_{i_1,l,i_2,l} = 1$, then Condition 2 implies $f_{i_1,j,0} - f_{i_2,j,0} \geqslant 0$ and $f_{i_1,l,0} - f_{i_2,l,0} \leqslant 0$.

We can directly apply Algorithm 1 to obtain draws uniformly from the admissible set $\mathcal{E}(\mathbf{\Sigma}, \mathbf{Q}, \mathcal{R}_0)$. In fact, the only modification one needs is to replace the sign restrictions set $\mathcal{S}_0$ with the ranking restrictions set $\mathcal{R}_0$ in the construction of the matrix $\mathbf{T}$. More specifically, we construct the $m \times n$ matrix $\mathbf{T}$ as follows: set $T_{ji} = +1$ if the i-th column of $\mathbf{R} = \mathbf{LQ}$ satisfies all the restrictions in $\mathcal{R}_0$ corresponding to j-th structural shock. If the negative of the i-th column of $\mathbf{R}$ satisfies all the inequalities in $\mathcal{S}_0$ corresponding to

j-th structural shock, set $T_{ji} = -1$; otherwise $T_{ji} = 0$. As before, $\mathbf{T}$ can be constructed with trivial computations. In addition, by Assumption 2, each column of $\mathbf{T}$ has at most one $+1$ or -1 since each column of $\mathbf{R}$ can satisfy (or violate) all the restrictions of at most one structural shock. The rest of the steps in Algorithm 1 remain exactly the same.

More generally, ranking restrictions include cases where two different structural shocks are involved. For example, one could impose $f_{i,j,0} \geqslant \lambda_{i,j,i,l} f_{i,l,0}$ for $\lambda_{i,j,i,l} > 1$, i.e., the response of the i-th variable to the j-th structural shock is larger than the response from the l-th structural shock. This type of restrictions can be accommodated by an extra accept-reject step. Specifically, one can first use Algorithm 1 to obtain a uniform draw that satisfies all other ranking restrictions. If this candidate draw also satisfies the additional ranking restrictions, we accept it; otherwise, we obtain another uniform draw until it is accepted. Similarly, this approach can be applied to cases when one wishes to impose sign or ranking restrictions on longer-horizon impulse responses.

3 Comparison of Computational Efficiency

In this section, we demonstrate the empirical performance of the proposed algorithm in various settings using the algorithm of Rubio-Ramirez, Waggoner, and Zha (2010) as a benchmark. In the first subsection, we consider an empirical example that involves a 15-variable VAR with over 40 sign and ranking restrictions. We compare both the speed and the estimated impulse responses from the two algorithms. In the second subsection, we further compare the computational efficiency of the proposed approach relative to the benchmark using a series of Monte Carlo simulations.

3.1 An Illustration of a 15-Variable VAR

We first illustrate the empirical performance of the proposed algorithm using a 15-variable VAR with over 40 sign and ranking restrictions to identify 5 structural shocks. As a comparison, we also use the algorithm of Rubio-Ramirez, Waggoner, and Zha (2010) to uniformly sample orthogonal matrices from the admissible set and compute the impulse responses.

More specifically, Furlanetto, Ravazzolo, and Sarferaz (2019) first use a 6-variable VAR to identify 5 structural shocks—demand, supply, monetary, investment and financial shocks—using a set of sign and ranking restrictions on the contemporaneous impact matrix. Chan (2022) augments their 6-variable system with 9 additional variables and sign restrictions. The variables and the structural restrictions are given in Table 1. All rows except the fourth present the sign restrictions on the contemporaneous impact matrix. The fourth row represents ranking restrictions: the entries denote the signs of the differential impacts on investment and output from each structural shock. For example, -1 in the demand column indicates that the impact from demand shocks on investment is smaller than the impact on output.

It is straightforward to see that this set of sign and ranking restrictions satisfies Assumption 2. More specifically, supply and monetary shocks can be distinguished from other shocks using Condition 1 in Assumption 2. In addition, demand shocks have a negative impact on the difference between investment and output, whereas the impacts from investment and financial shocks are positive. Hence, demand shocks can be distinguished from the other two shocks using Condition 3.

Table 1: Sign restrictions, ranking restrictions and identified shocks for the 15-variable VAR.

	Supply	Demand	Monetary	Investment	Financial
GDP	+1	+1	+1	+1	+1
GDP deflator	−1	+1	+1	+1	+1
3-month tbill rate	0	+1	−1	+1	+1
Investment/GDP	0	−1	0	+1	+1
S&P 500	+1	0	0	−1	+1
Spread	0	0	0	0	0
Spread 2	0	0	0	0	0
Credit/Real estate value	0	0	0	0	0
Mortgage rates	0	0	0	0	0
CPI	−1	+1	+1	+1	+1
PCE	−1	+1	+1	+1	+1
employment	0	0	0	0	0
Industrial production	+1	+1	+1	+1	+1
1-year tbill rate	0	+1	−1	+1	+1
DJIA	+1	0	0	−1	+1

Note: All restrictions are imposed on the response of a particular variable, except for investment/GDP, in which restrictions are imposed on linear inequalities of two responses.

As a benchmark, we use the algorithm of Rubio-Ramirez, Waggoner, and Zha (2010) to uniformly sample orthogonal matrices in conjunction with the posterior sampler of Chan (2022) designed for large VARs to obtain posterior draws of the model parameters. This approach requires approximately 3.6 billion draws from $\mathcal{U}(\mathbf{O}(n))$ to obtain 1,000 admissible draws, and the estimation takes about 6 days on a standard desktop. In contrast, the new algorithm requires only about 31,000 draws from $\mathcal{U}(\mathbf{O}(n))$ to obtain 1,000 admissible draws, and the entire exercise takes about 16 seconds.

Next, we empirically verify that the impulse responses obtained from the two algorithms are the same. In particular, Figure 1 reports the impulse responses of 6 variables to an one-standard-deviation financial shock, obtained using the algorithm of Rubio-Ramirez, Waggoner, and Zha (2010), and Figure 2 reports those from the proposed algorithm. As expected, the impulse responses obtained using the two algorithms are identical. Thus, these results highlight the utility of the proposed algorithm: it provides the same impulse responses but is several orders of magnitude more efficient than the benchmark.

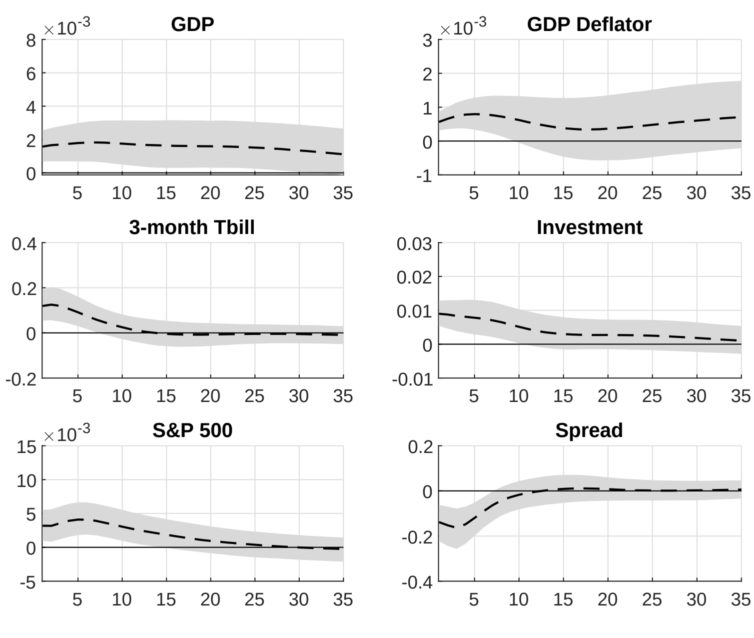

Figure 1: Impulse responses from a 15-variable VAR to an one-standard-deviation financial shock, obtained using the algorithm of Rubio-Ramirez, Waggoner, and Zha (2010).

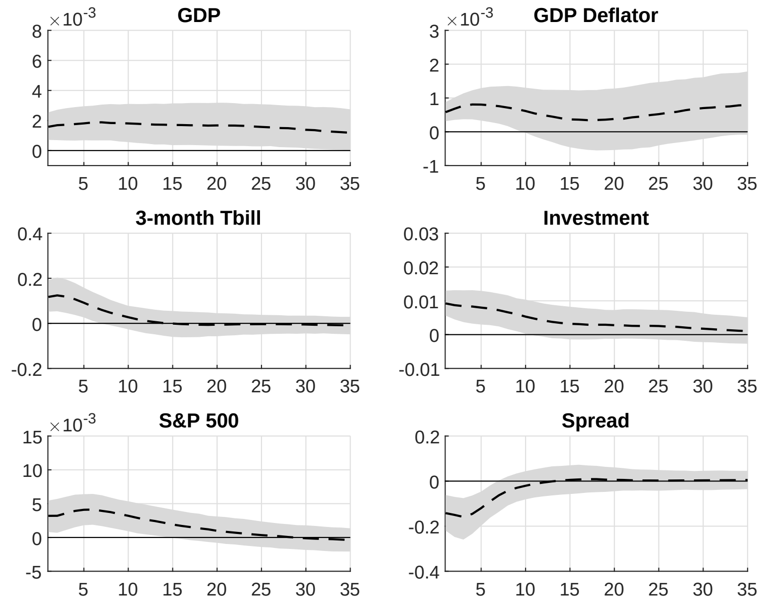

Figure 2: Impulse responses from a 15-variable VAR to an one-standard-deviation financial shock, obtained using the proposed algorithm described in Algorithm 1.

3.2 A Monte Carlo Study

Next, we conduct a series of Monte Carlo simulations to illustrate the empirical performance of the proposed method along a few dimensions of the model. More specifically, we consider different numbers of variables ($n = 10, 30, 50$) and structural shocks ($m = 5, 8$), while fixing sample size $T = 200$ and lag length $p = 5$ for all Monte Carlo simulations.

For each (n, m) combination, we generate a dataset from the VAR in (1)-(2) as follows. First, we draw the intercepts independently from the uniform distribution on the interval $(-1, 1)$, i.e., $\mathcal{U}(-1, 1)$. For the VAR coefficients, the diagonal elements of the first VAR coefficient matrix are iid $\mathcal{U}(0, 0.5)$ and the off-diagonal elements are from $\mathcal{U}(-0.2, 0.2)$; all other elements of the j-th ($j > 1$) VAR coefficient matrices are iid $\mathcal{N}(0, 0.1^2/j^2)$. Finally, to construct the impact matrix $\mathbf{B}_0$, we first draw the diagonal elements from iid $\mathcal{U}(0.5, 1.5)$, and the off-diagonal elements from iid $\mathcal{N}(0, 1)$. We then store them and change the signs of the elements in $\mathbf{B}_0$ to match the set of restrictions specified in each case.

Given a dataset, we then estimate the model using the proposed algorithm and the benchmark, together with the direct posterior sampler of Chan (2022) designed for large VARs. Each algorithm is run for 10,000 seconds, and we record the numbers of posterior draws and admissible draws (i.e., those posterior draws that satisfy all the structural restrictions). The results are reported in Table 2. The top panel refers to the case where only sign restrictions are used (and the set of sign restrictions satisfies Assumption 1). The bottom panel considers the case where three additional ranking restrictions are added.

Table 2: Numbers of posterior draws (in millions) and admissible draws obtained for an n-variable VAR with m shocks within 10,000 seconds using the proposed method and the algorithm of Rubio-Ramirez, Waggoner, and Zha (2010) (RWZ).

Top panel: sign restrictions only					
			$n = 10$	$n = 30$	$n = 50$
$m = 5$		# restrictions	25	35	40
	RWZ	Posterior draws ($\times 10^6$)	240	35	14
		Admissible draws	1,033	0	0
	Proposed method	Posterior draws ($\times 10^6$)	12	12	9
		Admissible draws	489,030	166,590	14,099
$m = 8$		# restrictions	40	50	60
	RWZ	Posterior draws ($\times 10^6$)	232	36	14
		Admissible draws	0	0	0
	Proposed method	Posterior draws ($\times 10^6$)	34	14	7
		Admissible draws	266,280	57,804	2,460
Bottom panel: 3 additional ranking restrictions					
			$n = 10$	$n = 30$	$n = 50$
$m = 5$		# restrictions	28	38	43
	RWZ	Posterior draws ($\times 10^6$)	258	34	13
		Admissible draws	115	0	0
	Proposed method	Posterior draws ($\times 10^6$)	47	18	8
		Admissible draws	310,970	14,230	1,214
$m = 8$		# restrictions	43	53	63
	RWZ	Posterior draws ($\times 10^6$)	260	34	14
		Admissible draws	0	0	0
	Proposed method	Posterior draws ($\times 10^6$)	37	13	6
		Admissible draws	99,607	2,525	1,000

It is clear from these results that the proposed method is much more efficient compared to the benchmark. Furthermore, when n or m grows, the differences in performance between the two methods become more apparent. In fact, in many cases with large n or large m, the sampling efficiency of the benchmark deteriorates so quickly that it becomes infeasible. In contrast, the proposed method remains capable of obtaining a large number of admissible draws for large n and m in a reasonable amount of time.

4 A 35-Variable VAR of the US Economy

To showcase the usefulness of the proposed algorithm, we consider an application that involves a 35-variable VAR with sign and ranking restrictions to identify 8 structural shocks, namely, demand, investment, financial, monetary policy, government spending, technology, labor supply and wage bargaining. The list includes many standard macroeconomic and financial variables, such as national accounts variables, various inflation indexes and interest rates, labor market variables, oil and stock prices. These variables are broadly similar to those used in Crump, Eusepi, Giannone, Qian, and Sbordone (2021) and are closely monitored by the Federal Reserve Staff and professional forecasters.

There are several reasons in favor of using a large set of macroeconomic and financial variables in structural analysis. First, a large system provides a convenient and unified framework to investigate the impacts of multiple structural shocks simultaneously. In particular, it allows the researcher to tease out the impacts of different structural shocks—such as different types of demand and supply shocks—and their individual contributions to macroeconomic fluctuations.

Second, it mitigates the concern of informational deficiency of using a limited information set, as pointed out in a series of influential papers by Hansen and Sargent (1991) and Lippi and Reichlin (1993, 1994). By using a larger set of relevant variables, one can close the gap between the set of variables considered by the economic agent and that considered by the econometrician, thus alleviating the concern of non-fundamentalness (see, e.g., Gambetti, 2021, for a recent review).

Third, as argued in Loria, Matthes, and Wang (2022), the mapping from variables in an economic model to the data is typically not unique. For example, one could match the economic variable 'inflation' to data based on the CPI, PCE, or the GDP deflator. One natural way to avoid an arbitrary choice is to include multiple data series corresponding to the same economic variable in the analysis.

The list of variables and the structural restrictions are given in Table 3.[2] The top part of the table lists the sign restrictions whereas the lower part lists the ranking restrictions. For example, the row labeled 'Government spending/GDP' lists the signs of the differences in impacts on government spending and GDP from each structural shock. In particular, the +1 in the government spending column indicates that the impact from government spending shocks on government spending is larger than the impact on GDP. It can be easily verified that the set of restrictions in Table 3 satisfies Assumption 2.

Table 3: Sign restrictions, ranking restrictions and identified shocks for the 35-variable VAR.

Sign restrictions	Demand	Investment	Financial	Monetary	Government spending	Technology	Labor supply	Wage bargaining
GDP	+1	+1	+1	−1	+1	+1	+1	+1
Personal consumption expenditure	0	0	0	0	0	+1	0	0
Residential investment	0	0	0	0	0	0	0	0
Nonresidential investment	0	0	0	0	0	+1	0	0
Exports	0	0	0	0	0	0	0	0
Imports	0	0	0	0	0	0	0	0
Government spending	0	0	0	0	+1	0	0	0
Federal budget surplus/deficit	0	0	0	0	−1	0	0	0
Federal tax receipts	0	0	0	0	+1	0	0	0
GDP deflator	+1	+1	+1	−1	+1	−1	−1	−1
PCE index	+1	+1	+1	−1	+1	−1	−1	−1
PCE index less food & energy	+1	+1	+1	−1	+1	−1	−1	−1
CPI index	+1	+1	+1	−1	+1	−1	−1	−1
CPI index less food & energy	+1	+1	+1	−1	+1	−1	−1	−1
Hourly wage	0	0	0	0	0	+1	−1	−1
Labor productivity	0	0	0	0	0	+1	0	0
Utilization-adjusted TFP	0	0	0	0	0	+1	0	0
Employment	0	0	0	−1	0	0	0	0
Unemployment rate	−1	−1	−1	+1	−1	−1	+1	−1
Industrial production index	+1	+1	+1	−1	0	0	0	0
Capacity utilization	+1	+1	+1	−1	0	0	0	0
Housing starts	0	0	0	0	0	0	0	0
Disposable income	0	0	0	0	0	0	0	0
Consumer sentiment	0	0	0	0	0	0	0	0
Fed funds rate	+1	+1	+1	+1	+1	0	0	0
3-month tbill rate	+1	+1	+1	+1	+1	0	0	0
2-year tnote rate	0	0	0	+1	0	0	0	0
5-year tnote rate	0	0	0	+1	0	0	0	0
10-year tnote rate	0	0	0	+1	0	0	0	0
Prime rate	+1	+1	+1	+1	+1	0	0	0
Aaa corporate bond yield	0	0	0	+1	0	0	0	0
Baa corporate bond yield	0	0	0	+1	0	0	0	0
Trade-weighted US$ index	0	0	0	0	0	0	0	0
S&P 500	0	−1	+1	−1	0	0	0	0
Spot oil price	0	0	0	0	0	0	0	0
Ranking restrictions								
Nonresidential investment/GDP	−1	+1	+1	0	0	0	0	0
Government spending/GDP	−1	−1	−1	0	+1	0	0	0

[2]Most variables are transformed by taking logs and multiplying 100, while others such as interest rates and unemployment rates are not transformed and are in percentages.

We use the asymmetric conjugate prior and the direct sampling approach in Chan (2022) to obtain posterior draws under the 35-variable VAR. The key advantage of the asymmetric conjugate prior is that it allows cross-variable shrinkage—i.e., shrinking coefficients on lags of other variables more aggressively to 0 than those on own lags—and at the same time it admits a closed-form expression of the marginal likelihood. In addition, since the prior is conjugate, one can directly sample independent draws from the posterior distribution instead of using MCMC methods.

We obtain the values of the optimal shrinkage hyperparameters on the VAR coefficients by maximizing the marginal likelihood of the model. Then, we use Algorithm 1 to obtain 1,000 admissible draws that satisfy all the sign and ranking restrictions. For this 35-variable VAR with over 100 sign and ranking restrictions, the entire exercise takes about 14 minutes and requires 557,000 draws from $\mathcal{U}(\mathbf{O}(n))$.

Figures 3–5 report the impulse responses of 6 selected variables to the (one-standard-deviation) demand, investment and financial shocks. As expected, these demand-type structural shocks raise output, short-term interest rate and inflation, while lowering both unemployment rate and real wage, at least in the short-run. Compared to the generic demand shock, both investment and financial shocks have a more substantive impact on nonresidential investment.

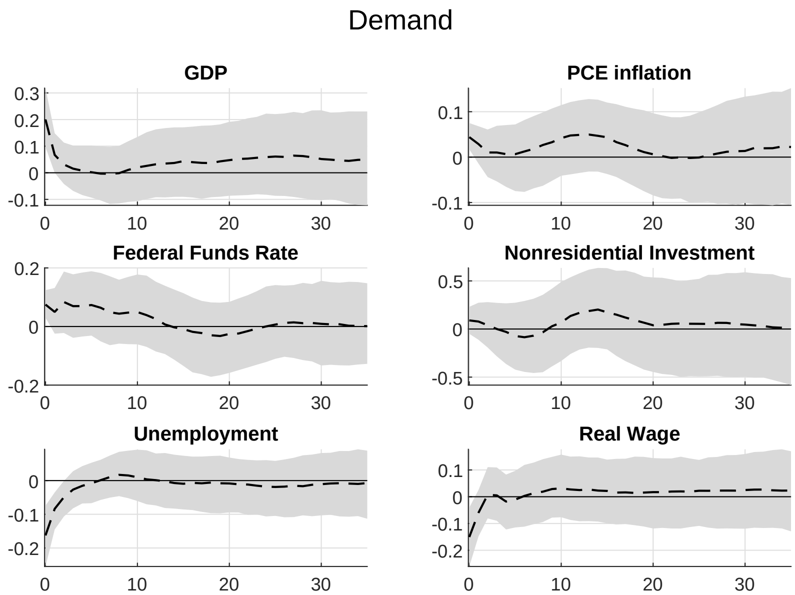

Figure 3: Impulse responses from a 35-variable VAR to an one-standard-deviation demand shock.

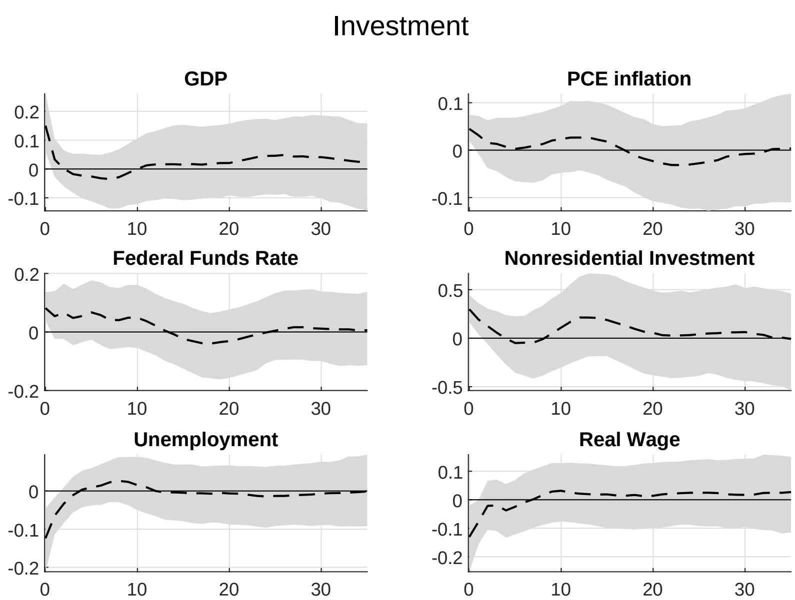

Figure 4: Impulse responses from a 35-variable VAR to an one-standard-deviation investment shock.

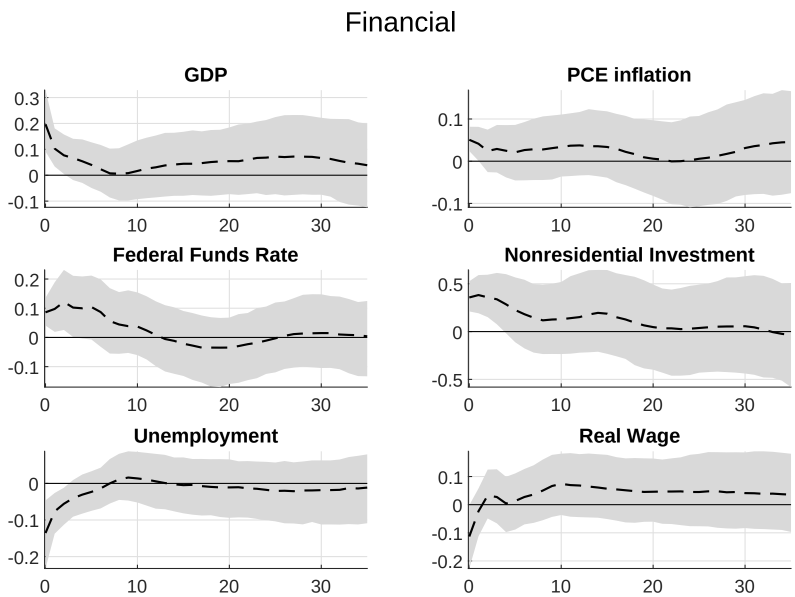

Figure 5: Impulse responses from a 35-variable VAR to an one-standard-deviation financial shock.

Next, Figures 6 and 7 plot the impulse responses of the same variables to the monetary policy shock and the government spending shock. A contractionary monetary policy shock depresses output and inflation, while raising the unemployment rate and the real wage. In contrast, an expansionary government spending shock mostly raises inflation and short-term interest rate, and has negligible effects on output, unemployment or the real wage.

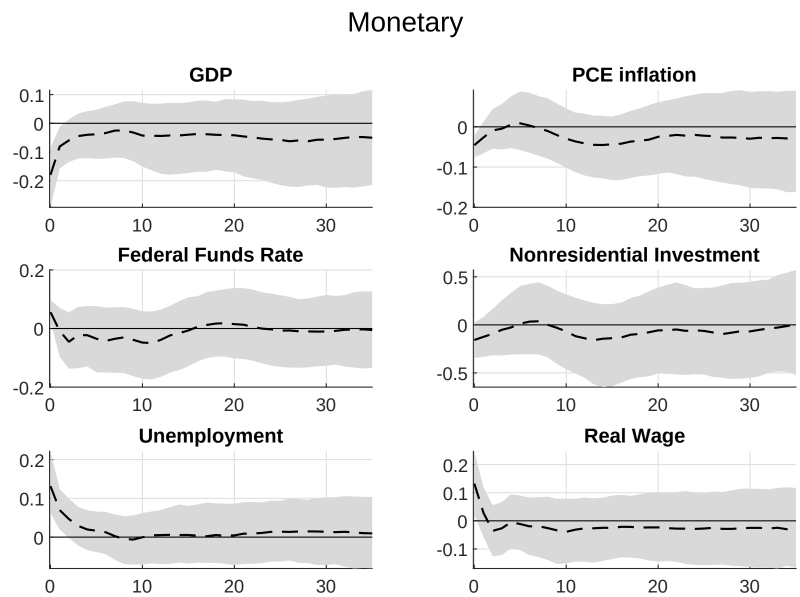

Figure 6: Impulse responses from a 35-variable VAR to an one-standard-deviation monetary policy shock.

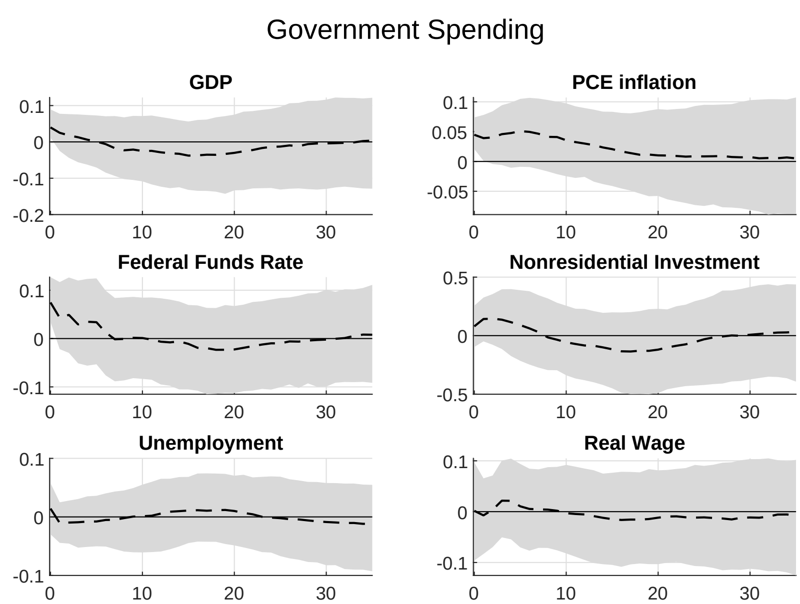

Figure 7: Impulse responses from a 35-variable VAR to an one-standard-deviation government spending shock.

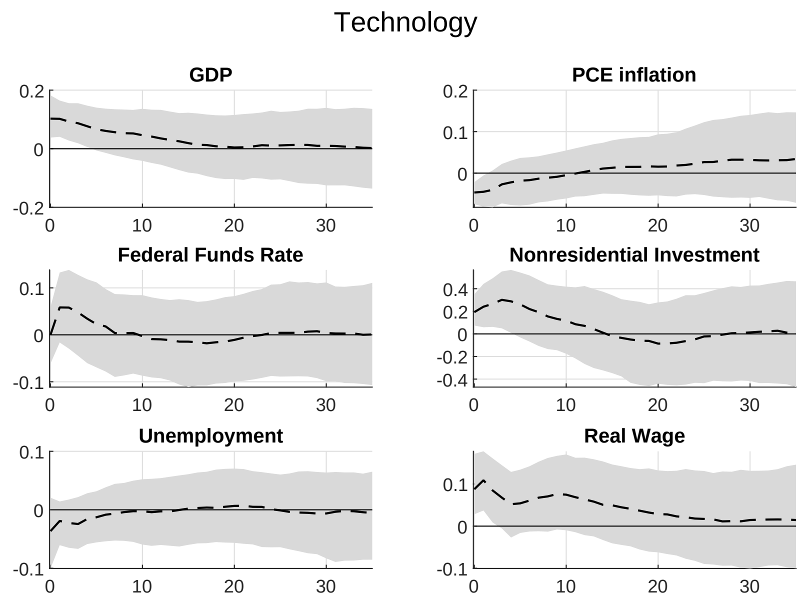

Figure 8: Impulse responses from a 35-variable VAR to an one-standard-deviation technology shock.

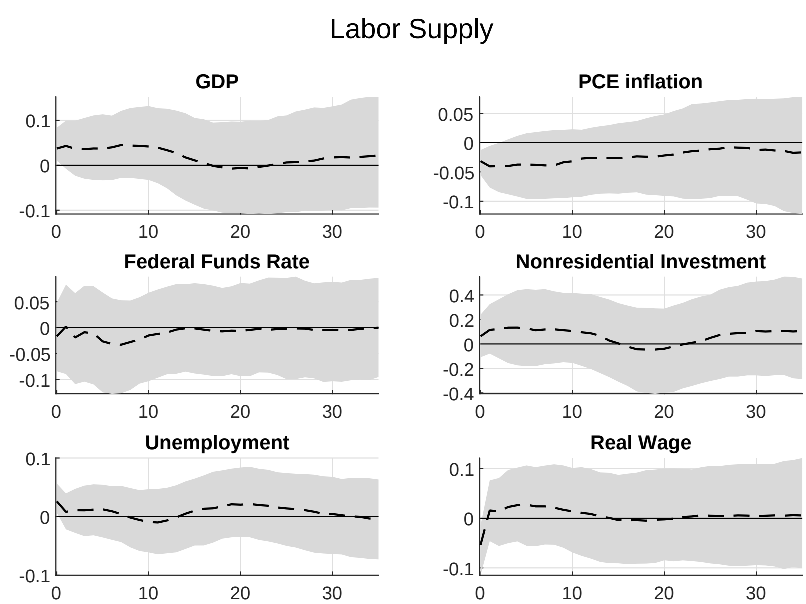

Figure 9: Impulse responses from a 35-variable VAR to an one-standard-deviation labor supply shock.

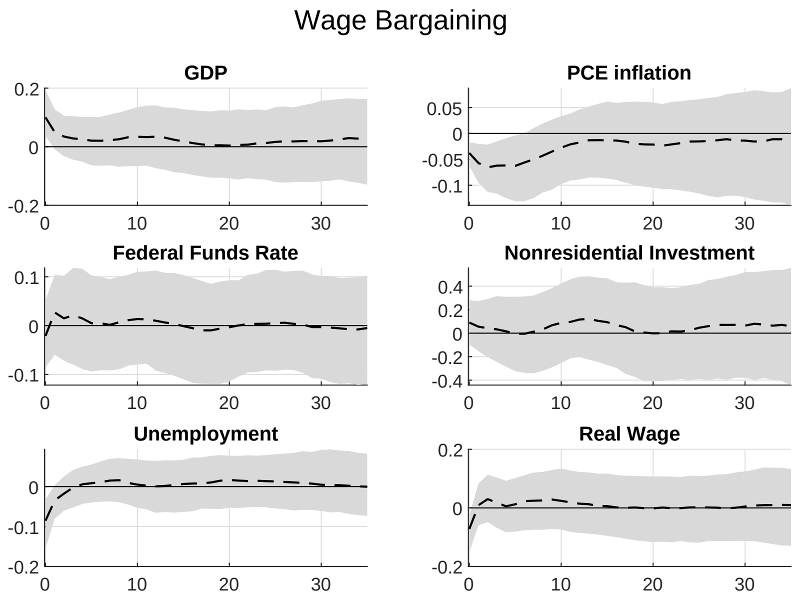

Figure 10: Impulse responses from a 35-variable VAR to an one-standard-deviation wage bargaining shock.

Finally, Figures 8–10 report the impulse responses of the 6 variables to the 3 supply-type structural shocks: technology, labor supply and wage bargaining shocks. While all 3 supply-type structural shocks raise output and depress inflation, the technology shock has the largest impact on these two variables. In addition, the technology shock substantially increases real wage over a relatively long horizon, whereas the other two structural shocks have transient and negligible impacts on real wage.

Overall, this application demonstrates that it is practical to study the impacts of multiple structural shocks jointly in a large VAR. Using a large number of sign and ranking restrictions to identify different structural shocks, we are able to disentangle their differential effects on key macroeconomic variables.

5 Concluding Remarks and Future Research

Two recent developments have motivated our paper: the recognition of the need to include a large number of variables in structural analysis and the desire to use more credible structural restrictions to identify structural shocks. In response to these developments, we have introduced an efficient approach for estimating large VARs identified using a large number of sign and ranking restrictions on the impulse responses. We showed that the new approach is several orders of magnitude more efficient than the benchmark, reducing the computational time from days to seconds. We illustrated the methodology using a 35-variable VAR with sign and ranking restrictions to identify 8 structural shocks.

For future research, it would be useful to extend the proposed algorithms to impose both inequality and zero restrictions (Arias, Rubio-Ramírez, and Waggoner, 2018), where the latter may arise in proxy VARs (Caldara and Herbst, 2019). It would also be interesting to incorporate richer prior information on the impact matrix or, more generally, impulse responses, as advocated in Baumeister and Hamilton (2015) and Bruns and Piffer (2023).

Appendix A: Proof of Proposition

In this appendix we provide a proof of the proposition stated in the main text.

Proof of Proposition 1. Let $\mathbf{L}$ denote the lower triangular Cholesky factor of $\mathbf{\Sigma}$ such that $\mathbf{\Sigma} = \mathbf{L}\mathbf{L}'$, and $\mathbf{Q} \sim \mathcal{U}(\mathbf{O}(n))$. Recall that $\mathcal{E}(\mathbf{\Sigma}, \mathbf{Q})$ consists of all the permutations and sign switches of the columns of $\mathbf{LQ}$. That is, an element $\mathbf{E} \in \mathcal{E}(\mathbf{\Sigma}, \mathbf{Q})$ can be represented as $\mathbf{E} = \mathbf{LQPD}$, where $\mathbf{P}$ is an n-dimensional permutation matrix and $\mathbf{D}$ is a diagonal matrix with elements ± 1. Since the Haar measure is invariant under right multiplication of $\mathbf{P}$ and $\mathbf{D}$, $\mathbf{QPD}$ is a uniform draw from the orthogonal group $\mathbf{O}(n)$. Next, recall that $\mathcal{E}(\mathbf{\Sigma}, \mathbf{Q}, \mathcal{S}_0)$ denotes the subset of elements in $\mathcal{E}(\mathbf{\Sigma}, \mathbf{Q})$ that satisfy all restrictions in $\mathcal{S}_0$. Step 3 of Algorithm 1 uniformly obtains an element $\widetilde{\mathbf{L}}$ in $\mathcal{E}(\mathbf{\Sigma}, \mathbf{Q}, \mathcal{S}_0)$, which can be represented as $\widetilde{\mathbf{L}} = \mathbf{LQPD}$ for some permutation matrix $\mathbf{P}$ and diagonal matrix $\mathbf{D}$ with elements ± 1. Hence, $\mathbf{Q}^* = \mathbf{QPD} \sim \mathcal{U}(\mathbf{O}(n))$ and $\widetilde{\mathbf{L}} = \mathbf{LQ}^*$. □

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